The Standard Model of Particle Physics

The Standard Model is a theory of the strong, weak and electromagnetic forces, formulated in the language of quantum gauge field theories, and of the elementary particles that take part in these interactions. It does, however, not include gravity. Interactions are mediated by the exchange of virtual particles.

FORCE	RELATIVE STRENGTH	RANGE
Strong (nuclear)	1	10^{-15} m
Weak (radioactive decay)	10^{-6}	10^{-18} m
Electromagnetic	α (10^{-2})	infinite
Gravitational	10^{-38}	infinite

Particle Content

Matter particles:

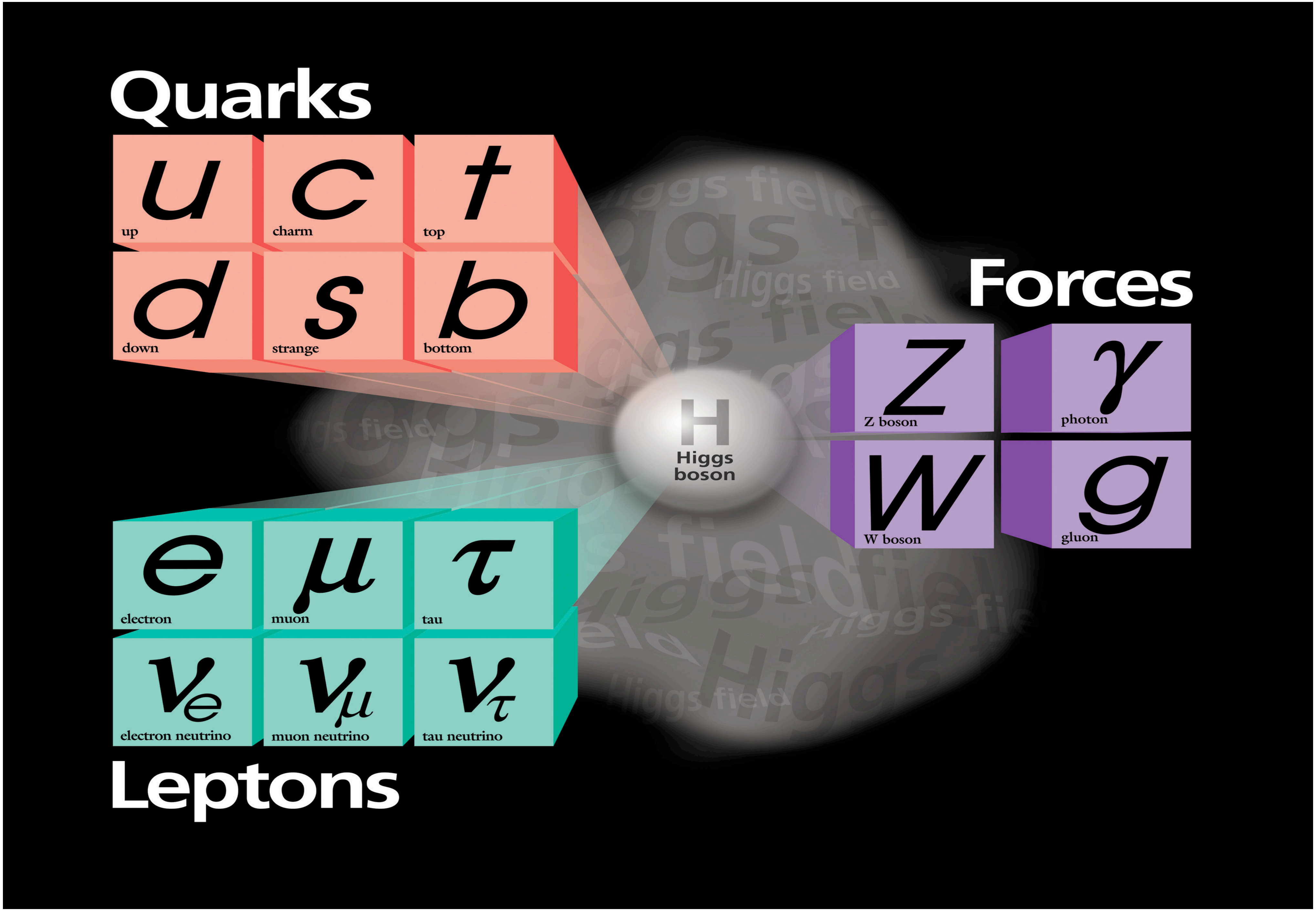
Fermions (half-integer spin, s = ½ħ) and their antiparticles.
There are 3 families (generations) of fermion fields, which are identical except for their masses. Fermions come as leptons and quarks.

Mediator particles:

Gauge bosons (integer spin, s = 1ħ).
There are 3 types of gauge bosons, corresponding to the 3 interactions described by the Standard Model.

Higgs particle:

Needed to explain that the symmetries of the electroweak theory are broken to the residual gauge symmetry of QED. Particles that interact with the Higgs field cannot propagate at the speed of light and acquire masses through coupling to the Higgs boson (s = 0ħ).



Quarks
u
up
c
charm
t
top
d
down
s
strange
b
bottom
Forces
Z
Z boson
γ
photon
W
W boson
g
gluon
H
Higgs boson
Higgs field
e
electron
μ
muon
τ
tau
νe
electron neutrino
νμ
muon neutrino
ντ
tau neutrino
Leptons

History of the Standard Model

1964 Quarks (u,d,s) are proposed by M. Gell-Mann and G. Zweig.

1964 The Higgs mechanism is developed by R. Brout, F. Englert, P. Higgs, G. Guralnik, C. Hagen, T. Kibble

1965 Color is proposed by O. W. Greenberg, M. Y. Han, Y. Nambu.

1967 S. Weinberg, Sh. Glashow and A. Salam create the electroweak theory, unifying the electromagnetic and weak nuclear forces (Nobel prize in 1979), and incorporate the Higgs mechanism to generate mass.

1969 J. Friedman, H. Kendall, R. Taylor find substructure of the proton (first evidence of quarks) in a deep elastic scattering experiment.

1970 GIM mechanism: fourth quark (c) allows a theory that suppresses flavor-changing neutral currents, mediated by the Z boson.

1970 Formulation of a quantum theory of the strong interaction (Quantum Chromodynamics, QCD) by H. Fritzsch and M. Gell-Mann.

1971 Renormalizability of Yang-Mills theories with spontaneous symmetry breaking (G. t'Hooft, M. Veltman)

1973 Asymptotic freedom by D. Politzer, D. Gross, F. Wilczek.

History of the Standard Model

1974 The Standard Model of particle physics is presented in its modern form by J. Iliopoulos.

1974 The charm quark is observed at SLAC (B. Richter et al.) and at Brookhaven (S. Ting et al.), through the discovery of the J/ψ.

1975 Evidence of the tau lepton is found at SLAC (M. Perl et al.).

1977 Evidence of the bottom quark (proposed in 1973 by M. Kobayashi, T. Maskawa) is found at Fermilab (L. Lederman et al.).

1983 The W and Z bosons, mediators of the weak-force, are discovered at CERN (C. Rubbia et al.).

1995 Evidence for the top quark, the final undiscovered quark, is found at Fermilab.

2000 The tau neutrino, the last missing lepton, is observed at Fermilab's DONUT experiment.

Today The search for the Higgs particle (and violations of the Standard Model…) is on!

Leptons

There are six leptons (and their antiparticles), classified according to their lepton number (L_e, L_μ, L_τ) and their electric charge (Q).

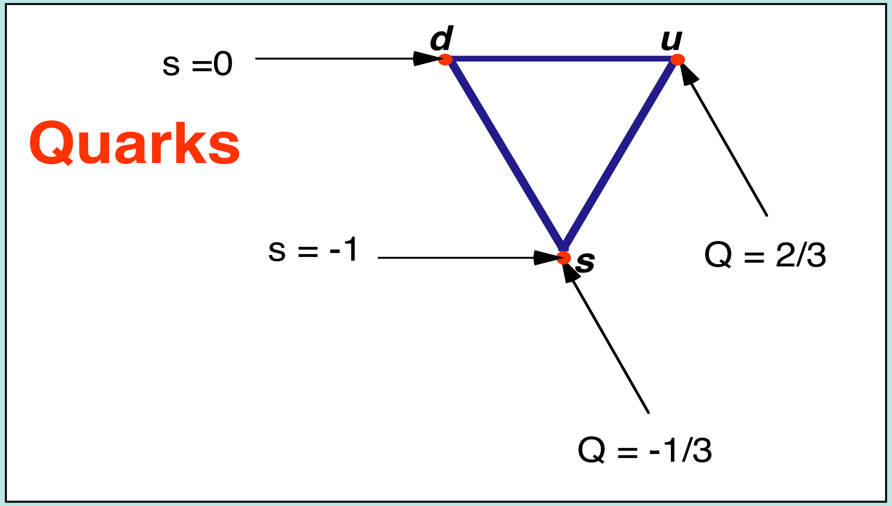
Lepton	L_e	L_μ	L_τ	Q	Mass
e^-	+1	0	0	-1	0.511 MeV
$\bar{\nu}_e$	-1	0	0	0	< 3 eV
e^+	-1	0	0	+1	0.511 MeV
ν_e	+1	0	0	0	< 3 eV
μ^-	0	+1	0	-1	105.7 MeV
$\bar{\nu}_\mu$	0	-1	0	0	< 0.19 MeV
μ^+	0	-1	0	+1	105.7 MeV
ν_μ	0	+1	0	0	< 0.19 MeV
τ^-	0	0	+1	-1	1.777 GeV
$\bar{\nu}_\tau$	0	0	-1	0	< 18.2 MeV
τ^+	0	0	-1	+1	1.777 GeV
ν_τ	0	0	+1	0	< 18.2 MeV

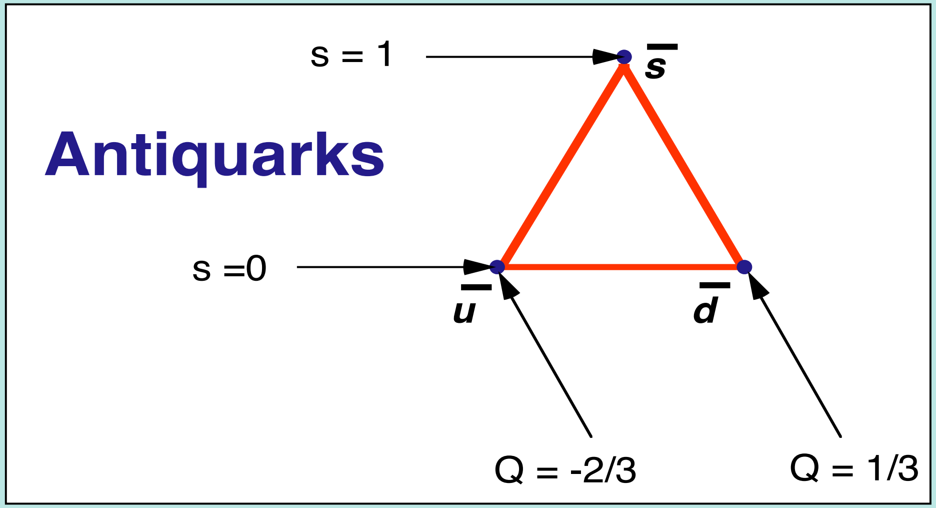
Quarks

There are six quarks (and six antiquarks), in three "colors" (and "anticolors"), with baryon number B = ±1/3 and fractional electric charges.

Quark	B	Q	Mass
u	1/3	2/3	1.5 - 3.3 MeV
$\overline{u}$	-1/3	-2/3	1.5 - 3.3 MeV
d	1/3	-1/3	3.5 - 6 MeV
$\overline{d}$	-1/3	1/3	3.5 - 6 MeV
c	1/3	2/3	$1.27^{+0.07}_{-0.11}$ GeV
$\overline{c}$	-1/3	-2/3	$1.27^{+0.07}_{-0.11}$ GeV
s	1/3	-1/3	105^{+25}_{-35} MeV
$\overline{s}$	-1/3	1/3	105^{+25}_{-35} MeV
t	1/3	2/3	171.3 ± 1.1 ± 1.2 GeV
$\overline{t}$	-1/3	-2/3	171.3 ± 1.1 ± 1.2 GeV
b	1/3	-1/3	$4.20^{+0.17}_{-0.07}$ GeV
$\overline{b}$	-1/3	1/3	$4.20^{+0.17}_{-0.07}$ GeV

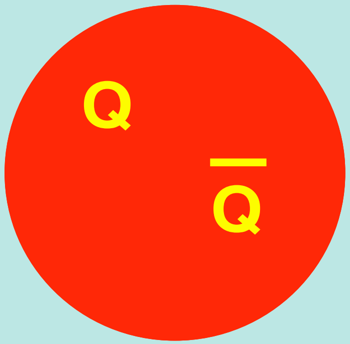
Quark Model

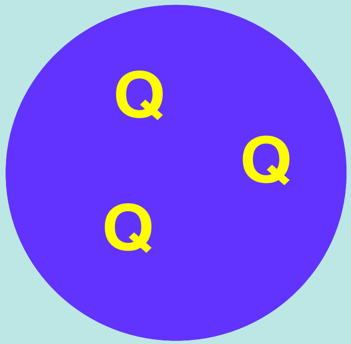




S: Strangeness (S = - 1 for s quark)

Mesons are made of quark-antiquark pairs, baryons consist of 3 quarks.





Strong Isospin

Proton and neutron have about the same mass. Therefore it was convenient to order them into a doublet:

$$\begin{pmatrix} p \\ n \end{pmatrix}$$

In other words, they are the same particle with respect to the strong interaction (same "strong isospin" I) but with a different third component of strong isospin (I_3).

$$|p\rangle = \left|\tfrac{1}{2}, +\tfrac{1}{2}\right\rangle \qquad |n\rangle = \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

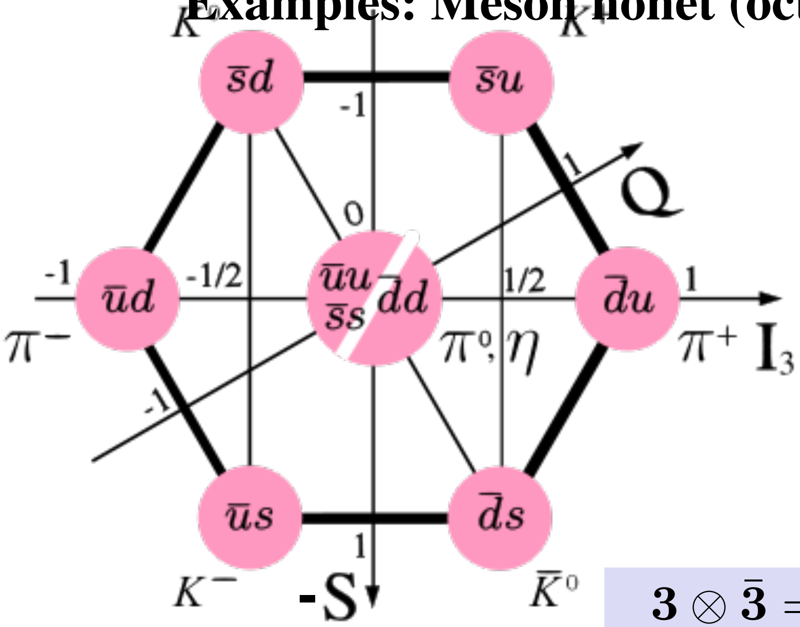
$$|\pi^+\rangle = |1, +1\rangle,\ |\pi^0\rangle = |1, 0\rangle, \text{ and } |\pi^-\rangle = |1, -1\rangle$$

Going to quarks, isopin is a quantum number that distinguishes flavor. Each of the 3 lighter quarks has a different orientation of I_3:
$I_3(u) = ½$, $I_3(d) = -½$, $I_3(s) = 0$

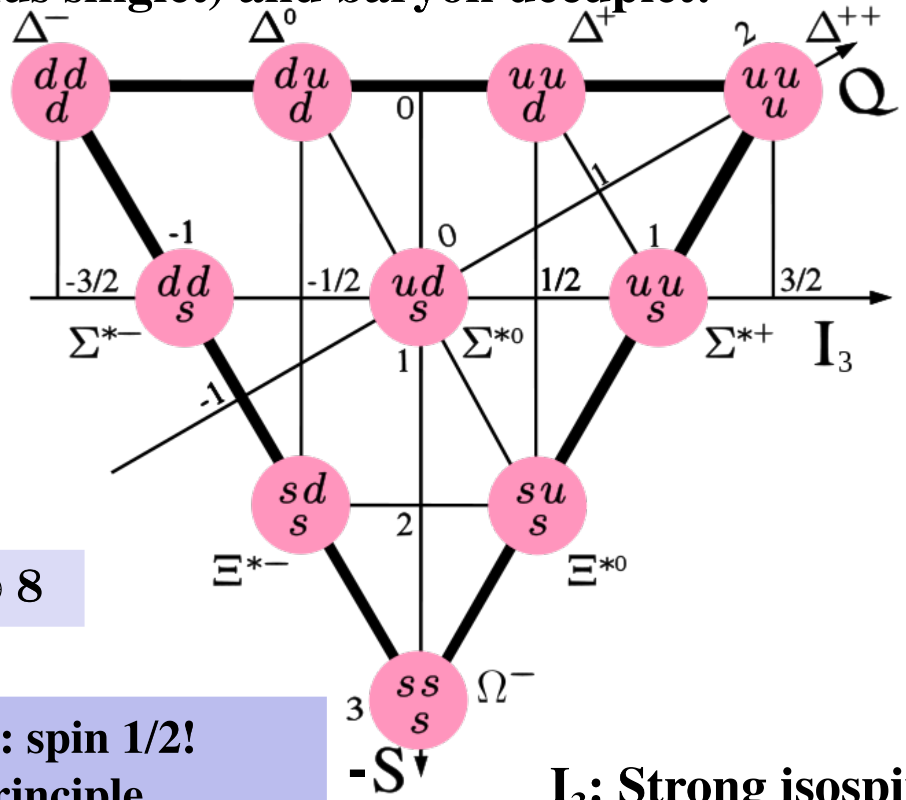
Many particle properties can be related to specific symmetries.

Hadrons: Mesons and Baryons

Examples: Meson nonet (octet plus singlet) and baryon decuplet.



$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$



$$\pi^0 : (u\bar{u} - d\bar{d})/\sqrt{2}$$
$$\eta : (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$$
$$\eta' : (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$$

Quarks: spin 1/2!
Pauli principle
-> COLOR
(O.W. Greenberg et al.)

I_3: Strong isospin (3rd component),
S: Strangeness

Interactions

Interaction	Mediator	Mass	Acts on
Electromagnetic	γ Photon	0	Particle with electric charge
Weak	W^+, W^-, Z^0 Intermediate vector bosons	80 GeV 90 GeV	Left-handed particle or right-handed antiparticle with flavor
Strong	$g_1, \ldots, g_8$ Gluons	0	Particle with color

Neutrinos have only weak interactions, charged leptons experience weak and electromagnetic interactions, quarks have strong, weak and electromagnetic interactions.
In the classical Standard Model, neutrinos are massless.
Fermions come in left-handed weak isospin doublets and right-handed singlets, quarks come in color triplets.

Left-handed doublets, right-handed singlets

Left-handed chiral fermion doublets and right-handed singlets:

$$\mathbf{L} = \begin{pmatrix} \nu \\ l \end{pmatrix}_L \qquad \mathbf{R} = l_R, (\nu_l)_R \qquad (l = e, \mu, \tau)$$

If neutrinos are massless, as in the classical Standard Model, we would only have R = l_R.

T_3 … 3rd component of weak isospin

$$T_{3L} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \qquad T_{3R} = 0$$

Similarly for quarks: $\begin{pmatrix} u \\ d \end{pmatrix}_L, u_R, d_R \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L, c_R, s_R \qquad \begin{pmatrix} t \\ b \end{pmatrix}_L, b_R, t_R$

Only left-handed chiral particle states (or right-handed chiral antiparticle states) take part in the weak interaction.

Weak Isospin

Leptons and quarks have another quantum number: weak isospin.

Weak isospin connects quark and lepton doublets of left-handed particles, in each generation. Left-handed fermions (fermions with negative chirality) have $T = ½$ and can be grouped into doublets with $T_3 = \pm ½$ that behave the same under the weak interaction.

$T_3(u_L) = T_3(c_L) = T_3(t_L) = ½$, $T_3(d_L) = T_3(s_L) = T_3(b_L) = -½$
$T_3(e^-_L) = T_3(\mu^-_L) = T_3(\tau^-_L) = -½$, $T_3(\nu_{eL}) = T_3(\nu_{\mu L}) = T_3(\nu_{\tau L}) = ½$

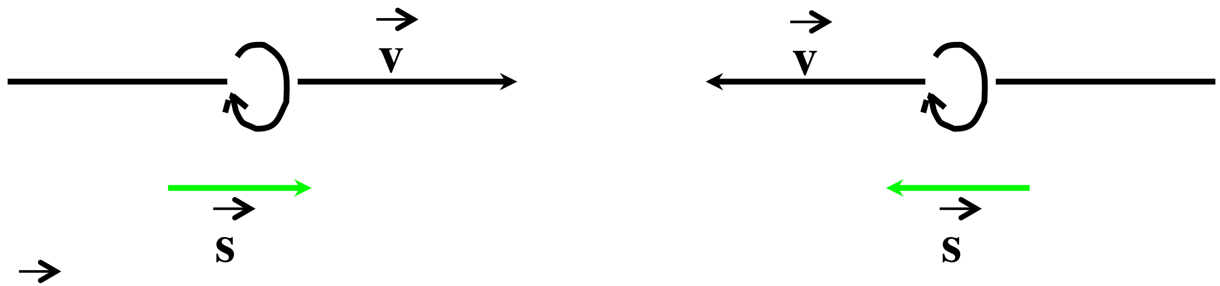
In analogy to the Gell-Mann-Nishijima formula ($Q = I_3 + Y/2$; $Y = B+S$):

$$\boxed{Y_W = 2\,(Q-T_3)}$$

Y_W … weak hypercharge

Helicity

Helicity (*h*) corresponds to the sign of the projection of spin onto the direction of motion. It is, however, not Lorentz invariant. This can be seen if the inertial system in the right-handed case moves at a speed faster than $\vec{v}$: h changes from +1 to -1.



$\vec{v} \parallel \vec{s}$: h = +1 ("right-handed") h = - 1 ("left-handed")

For a massless particle there is no inertial system that can move faster than the speed of light, therefore for such particles h is Lorentz invariant. For massless particles, helicity is also the same as chirality.

Spinors, Dirac Equation

Dirac equation for a free fermion with mass *m*: $(i\gamma^\mu\partial_\mu - m)\,\psi(x) = 0$

$\psi(x)$ ... 4-component Dirac spinor, $x = (t,x,y,z)$

$$\psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

4 fundamental solutions:

$$\psi_1 = u_1 e^{i(\overrightarrow{p}\,\overrightarrow{r} - Et)} \qquad \psi_3 = v_1 e^{-i(\overrightarrow{p}\,\overrightarrow{r} - Et)}$$
$$\psi_2 = u_2 e^{i(\overrightarrow{p}\,\overrightarrow{r} - Et)} \qquad \psi_4 = v_2 e^{-i(\overrightarrow{p}\,\overrightarrow{r} - Et)}$$

$$u_1 = N \begin{pmatrix} 1 \\ 0 \\ p_z/(|E|+m) \\ (p_x + ip_y)/(|E|+m) \end{pmatrix} \qquad u_2 = N \begin{pmatrix} 0 \\ 1 \\ (p_x - ip_y)/(|E|+m) \\ -p_z/(|E|+m) \end{pmatrix}$$

$$N = \sqrt{\tfrac{|E|+m}{2m}}$$
$$|E| = E_+ = |E_-|$$

$$v_1 = N \begin{pmatrix} p_z/(|E|+m) \\ (p_x + ip_y)/(|E|+m) \\ 1 \\ 0 \end{pmatrix} \qquad v_2 = N \begin{pmatrix} (p_x - ip_y)/(|E|+m) \\ -p_z/(|E|+m) \\ 0 \\ 1 \end{pmatrix}$$

u_1, u_2 describe a particle, v_1, v_2 an antiparticle.
The spin of u_1, v_1 is in +z-direction, the spin of u_2, v_2 in -z-direction.

Chirality

The eigenstates of the chirality operator γ^5 are defined as left-handed (u_L, v_L) and right-handed (u_R, v_R) chiral states:

$$\gamma^5 u_R = +u_R, \gamma^5 u_L = -u_L, \gamma^5 v_R = -v_R, \gamma^5 v_L = +v_L$$

Projection operators project out the chiral eigenstates:

$$P_R = \tfrac{1}{2}(1+\gamma^5) \qquad P_L = \tfrac{1}{2}(1-\gamma^5)$$

$$\begin{array}{llll} P_R u_R = u_R & P_R u_L = 0 & P_L u_R = 0 & P_L u_L = u_L \\ P_R v_R = 0 & P_R v_L = v_L & P_L v_R = v_R & P_L v_L = 0 \end{array}$$

P_R projects out right-handed particle states and left-handed antiparticle states.
We can write any spinor in the form of its chiral components:

$$\psi = \psi_R + \psi_L = \tfrac{1}{2}(1+\gamma^5)\psi + \tfrac{1}{2}(1-\gamma^5)\psi$$

Dirac and Pauli Matrices

Dirac (γ) matrices (4 x 4)

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} \underline{0} & \sigma_i \\ -\sigma_i & \underline{0} \end{pmatrix}$$

$$\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} \underline{0} & \underline{1} \\ \underline{1} & \underline{0} \end{pmatrix}$$

Pauli matrices (2 x 2)

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Group Structure of the Standard Model

In a gauge theory there is a group of transformations of the field variables (gauge transformations) that leaves the physics of the quantum field unchanged. This condition is called gauge invariance.

The Standard Model is a gauge theory. It is based on the symmetry group:

$$\mathbf{SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$$

The gauge symmetry is broken by the vacuum. The electroweak group is broken down to the electromagnetic subgroup through Spontaneous Symmetry Breaking (SSB) to:

$$\mathbf{SU(3)_C \otimes U(1)_{QED}}$$

SSB generates the masses of the weak gauge bosons, and gives rise to a scalar particle, the Higgs particle. The fermion masses are also generated through SSB.

Group Theory

Let us consider the transformation U of a wave function ψ:

$$\psi' = U\psi$$

If U is a continuous transformation, U has the form:

$$U = e^{i\Lambda} \qquad \Lambda \ldots \text{operator}$$

If Λ is a hermitian operator ($\Lambda = \Lambda^{+} = \Lambda^{*T}$), U is a unitary transformation:

$$U = e^{i\Lambda} \quad U^{+} = (e^{i\Lambda})^{*T} = e^{-i\Lambda^{*T}} = e^{-i\Lambda} \;\Rightarrow\; UU^{+} = e^{i\Lambda}\, e^{-i\Lambda} = 1$$

Remark: U is not a hermitian operator since $U \neq U^{+}$

Λ is called a *generator* of U.

The following four properties define a group:

1) **Closure: If A and B are elements of the group, $A \circ B$ is also an element.**
2) **Identity: For all group elements A: $I \circ A = A$.**
3) **Inverse: For each group element there is an inverse element such that $A \circ A^{-1} = I$.**
4) **Associativity: If A,B,C are group elements, $A \circ (B \circ C) = (A \circ B) \circ C$ are also elements.**

The group is *abelian* if the commutativity relation holds: $A \circ B = B \circ A$

The group is *special* if the determinant is det U = 1.

Remarks: Transformation with only one parameter Λ yields the unitary abelian group U(1).
The group SU(2) is an example of a non-abelian group.

Symmetries

Interactions among fundamental particles are described by symmetry principles. Every symmetry of nature yields a conservation law, every conservation law reveals an underlying symmetry (Noether's theorem). Examples:

Symmetry	Conservation law
Translation in time	Energy
Translation in space	Momentum
Rotation	Angular momentum
Gauge transformation	Charge

All fundamental interactions are invariant under local gauge transformations.

Dynamics among fundamental particles are described by a "Lagrange density" or "Lagrangian", which depends on the field and its derivative.

Lagrangian of a free fermion with mass *m*:

$$\mathcal{L}_0 = i\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) - m\bar{\psi}(x)\psi(x)$$

Global and Local Gauge Transformations

Transformation: $\psi \to U\psi$

U: Group of all unitary matrices. Simplest case: $U(1)$, $U = e^{i\theta}$, θ = *real constant*.

$\mathcal{L}_0$ is invariant under *global* $U(1)$ transformation:

$$\boxed{\psi(x) \quad \to \quad \psi'(x) = e^{i\theta}\psi(x)}$$

$$\mathcal{L}_0 \to ie^{-i\theta}\psi(x)\gamma^\mu e^{i\theta}\partial_\mu\psi(x) - me^{-i\theta}\bar{\psi}(x)e^{i\theta}\psi(x) = \mathcal{L}_0$$

If one allows the phase transformation to depend on space-time ($\theta = \theta(x)$), $\mathcal{L}_0$ is no longer invariant under this *local* transformation, because:

$$\partial_\mu\psi(x) \to e^{i\theta}\psi(x)\ (\partial_\mu + i\partial_\mu\theta)\ \psi(x)$$

This means that once a given phase convention has been adopted at a reference point, the same convention must be taken at all space points -> unnatural!

The gauge principle is the requirement that the $U(1)$ phase invariance should hold locally.

Covariant Derivative

Try to add extra piece to the Lagrangian, a new spin-1 field $A_\mu(x)$, transforming as:

$$A_\mu(x) \quad \rightarrow \quad A'_\mu(x) = A_\mu(x) - \tfrac{1}{e}\partial_\mu\theta$$

One also defines the covariant derivative:

$$\boxed{\mathcal{D}_\mu\psi(x) := \left[\partial_\mu + ieA_\mu(x)\right]\,\psi(x)}$$

The covariant derivative transforms just like the field itself:

$$\mathcal{D}_\mu\psi(x) \quad \rightarrow \quad (\mathcal{D}_\mu\psi)'(x) = e^{i\theta}\mathcal{D}_\mu\psi(x)$$

We achieve a Langrangian invariant under local gauge transformations:

$$\mathcal{L} = i\bar{\psi}(x)\gamma^\mu\mathcal{D}_\mu\psi(x) - m\bar{\psi}(x)\psi(x) = \mathcal{L}_0 - eA_\mu(x)\bar{\psi}(x)\gamma^\mu\psi(x)$$

Quantum Electrodynamics (QED)

The gauge principle has generated an interaction between the Dirac spinor and the gauge field A_μ.
To get the complete Langrangian for QED we have to add a kinetic term and in principle a mass term:

$$\mathcal{L}_{kin} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

electromagnetic field strength (tensor)

$$\mathcal{L}_{mass} = \frac{1}{2}m^2 A^\mu A_\mu$$

This term violates gauge invariance, therefore the photon mass must be 0!

$$\mathcal{L}_{QED} = \bar{\psi}(x)(i\gamma^\mu \mathcal{D}_\mu - m)\psi(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$$

$$\partial_\mu F^{\mu\nu} = J^\nu = e\bar{\psi}\gamma^\nu\psi$$

Maxwell equations

J^ν … fermion electromagnetic current

Lepton Anomalous Magnetic Moments

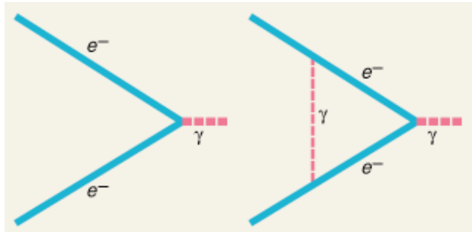
The most stringent experimental tests of QED come from the high-precision measurements of the electron (and muon) magnetic moments:

$$\mu_e = \frac{1}{2} g \mu_0 \qquad \mu_0 = \frac{e}{2m}$$

… Bohr's Magneton
$9 \cdot 10^{-24}\ \mathrm{JT}^{-1}$

"Anomaly" of the magnetic moment:

$$\boxed{a = \frac{g-2}{2}}$$



a_e arises entirely from virtual electrons and photons. These contributions are fully known to $O(\alpha^4)$; $\alpha = e^2/4\pi \approx 1/137$ is the QED coupling constant (fine structure constant).

$$a_e = (\ 1\ 159\ 652\ 180.85 \pm 0.76\ ) \cdot 10^{-12}$$

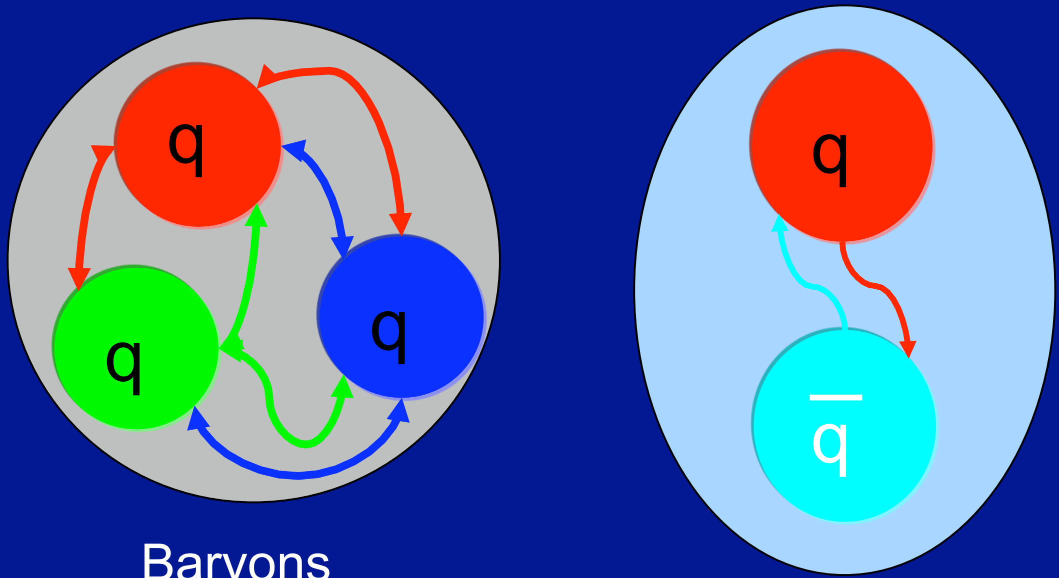
a_e provides also the most accurate determination of α:

$$\alpha^{-1} = 137.035\ 999\ 719 \pm 0.000\ 000\ 096$$

Quantum Chromodynamics

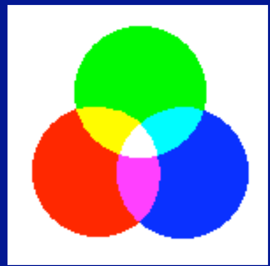
Besides the electric charge quarks also have a color charge. Gluons also have color charges, which are however not pure but mixed. The theory of the strong interaction is called quantum chromodynamics.

Color	Anticolor
RED	CYAN
BLUE	YELLOW
GREEN	MAGENTA



Baryons

Mesons



Introduction of Color

In order to satisfy Fermi-Dirac statistics, Quarks have 3 color degrees of freedom: $N_C = 3$ "(red, blue, green)".

$$\mathbf{q(r)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{q(b)} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \mathbf{q(g)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Example $\Delta^{++} = | u\uparrow u\uparrow u\uparrow >$ $\quad J^P = 3/2^+$

Angular momentum J

Parity P: symmetry under spatial (mirror) transformations $(x \to -x, y \to -y, z \to -z)$

The wave function of the Δ^{++} is totally symmetric without color:

$$\psi_{qqq} = \psi_{space}\, \psi_{spin}\, \psi_{flavor}$$

Restore asymmetry by introducing ψ_{color}, which is totally antisymmetric:

$$\psi_{qqq} = \psi_{space}\, \psi_{spin}\, \psi_{flavor}\, \psi_{color}$$

Color and Confinement

For baryons and mesons (quarks q_α, α = *1,2,3* for red, green, blue) the color term can be expressed as:

$$B = \frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}|q_\alpha q_\beta q_\gamma\rangle \qquad M = \frac{1}{\sqrt{3}}\delta^{\alpha\beta}|q_\alpha \bar{q}_\beta\rangle$$

$\varepsilon^{\alpha\beta\gamma}$ (Levi-Civita symbol) is +1 for even permutations of α,β,γ *(1,2,3; 2,3,1; 3,1,2),* -1 for odd permutations *(1,3,2; 3,2,1; 2,1,3)* , and 0 for $\alpha=\beta$ or $\beta=\gamma$ or $\gamma=\alpha$.

$\delta^{\alpha\beta}$ (Kronecker delta) is 1 for $\alpha=\beta$, and 0 for $\alpha\neq\beta$.

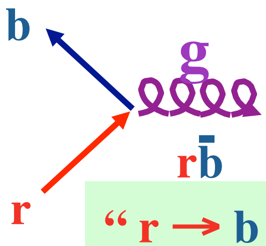
Summation over equal indices is implied.

Baryons and mesons appear only in color-singlet combinations.

We observe no free particles with non-zero color.
Free quarks can therefore not be observed, they are *confined* inside the hadrons, just like the gluons.

Gluons

Gluons can have mixed colors:



“ $r \rightarrow b + r\bar{b}$ ”

The red quark turns into a blue quark, emitting a red-antiblue gluon.

Are there 9 gluons? : $r\bar{r}, r\bar{b}, r\bar{g}, b\bar{r}, b\bar{b}, b\bar{g}, g\bar{r}, g\bar{b}, g\bar{g}$

In terms of color $SU(3)_C$ symmetry, these 9 states constitute a color octet (|1> … |8>) and a singlet (|9>):

$$
\begin{aligned}
|1\rangle &= (r\bar{b} + b\bar{r})/\sqrt{2} \\
|2\rangle &= -i\ (r\bar{b} - b\bar{r})/\sqrt{2} \\
|3\rangle &= (r\bar{r} - b\bar{b})/\sqrt{2} \\
|4\rangle &= (r\bar{g} + g\bar{r})/\sqrt{2} \\
|5\rangle &= -i\ (r\bar{g} - g\bar{r})/\sqrt{2} \\
|6\rangle &= (b\bar{g} + g\bar{b})/\sqrt{2} \\
|7\rangle &= -i\ (b\bar{g} - g\bar{b})/\sqrt{2} \\
|8\rangle &= (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6} \\
|9\rangle &= (r\bar{r} + b\bar{b} + g\bar{g})/\sqrt{3}
\end{aligned}
$$

Confinement requires that all naturally occurring particles are color singlets (color invariant), therefore the octet never appears as free particles. However, |9> is a singlet! Could it be the photon or another particle giving rise to a long-range force with strong coupling?

NO! Our world would be different…

Experimental Proof of Color

Measurement of the ratio of the total cross sections for e^+e^- annihilation into hadrons and muons:

$$R = \frac{\sigma\,(e^+e^- \rightarrow \text{hadrons})}{\sigma\,(e^+e^- \rightarrow \mu^+\mu^-)}$$

f … quark flavors u, d, s, c, b, t
N_C … number of color charges

Since the 3 color states have identical electric charges, the cross section for the production of quark pairs of a certain flavor type should be proportional to the number of color charges.

$$\sigma(e^+e^- \rightarrow q\bar{q}) = N_C\ (q_u^2 + q_d^2 + q_s^2 ...)\ \sigma(e^+e^- \rightarrow \mu^+\mu^-)$$

$$R_0 = \sigma(e^+e^- \rightarrow q\bar{q})/\sigma(e^+e^- \rightarrow \mu^+\mu^-) = N_C\ (q_u^2 + q_d^2 + q_s^2 ...)$$

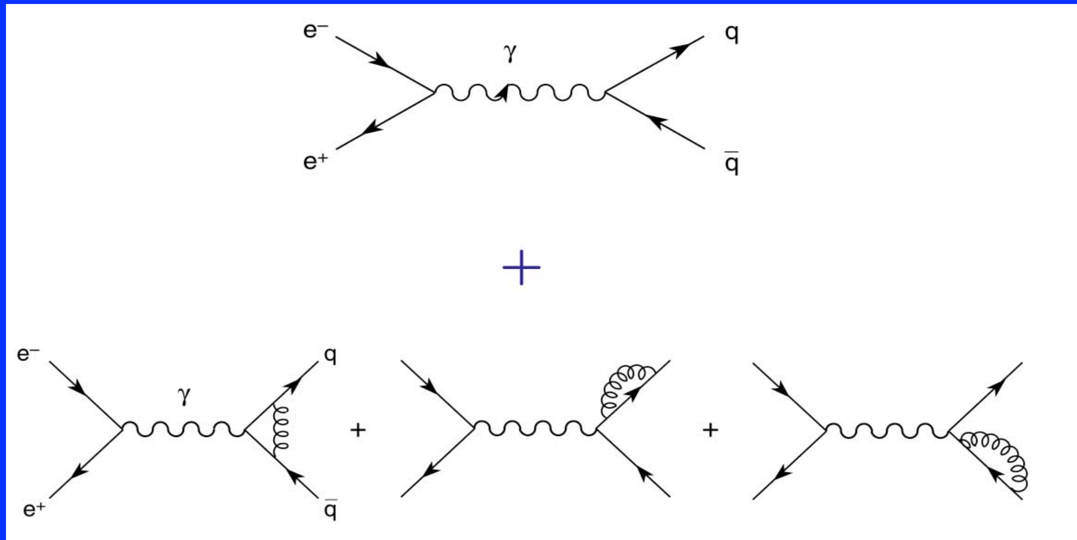
Taking into account higher orders (e.g. 3-jet events) yields:

$$R = R_0\ (1 + \alpha_s(Q^2)/\pi + ...)$$

Q^2 … momentum transfer

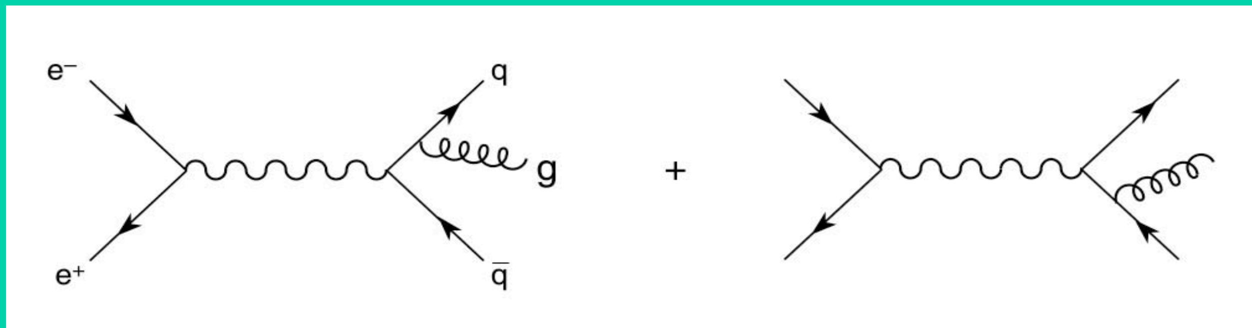
$$\sigma\,(e^+e^- \rightarrow \text{hadrons}) = \sigma\,(e^+e^- \rightarrow q\bar{q} + q\bar{q}g + q\bar{q}gg + q\bar{q}q\bar{q} + \ldots\ )$$

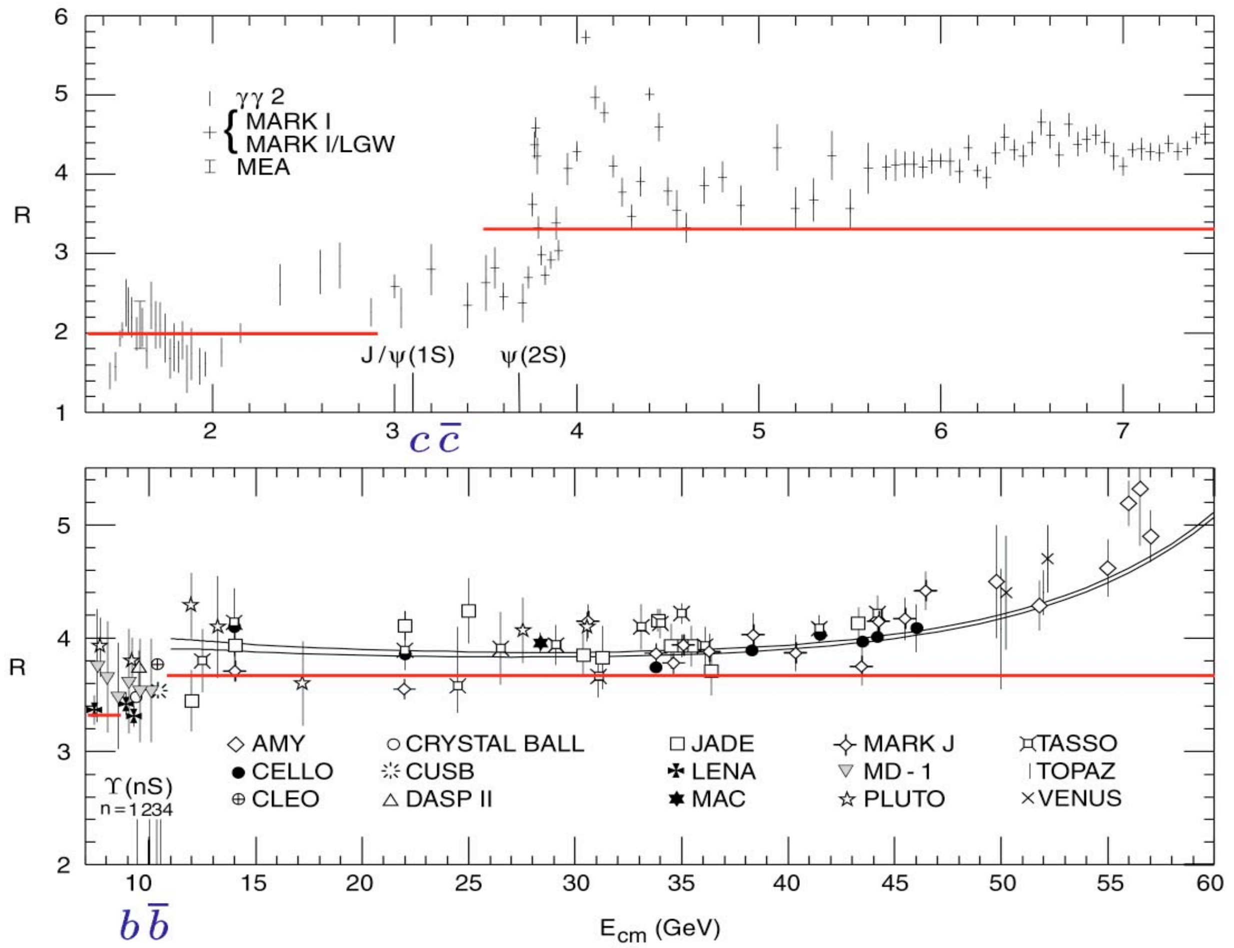
$q\bar{q}$



R is almost constant, as $e^+e^- \rightarrow q\bar{q}$ dominates.

$q\bar{q}g$





u, d, s: $R_0 = 3\,(q_u^{\,2} + q_d^{\,2} + q_s^{\,2}) = 2$

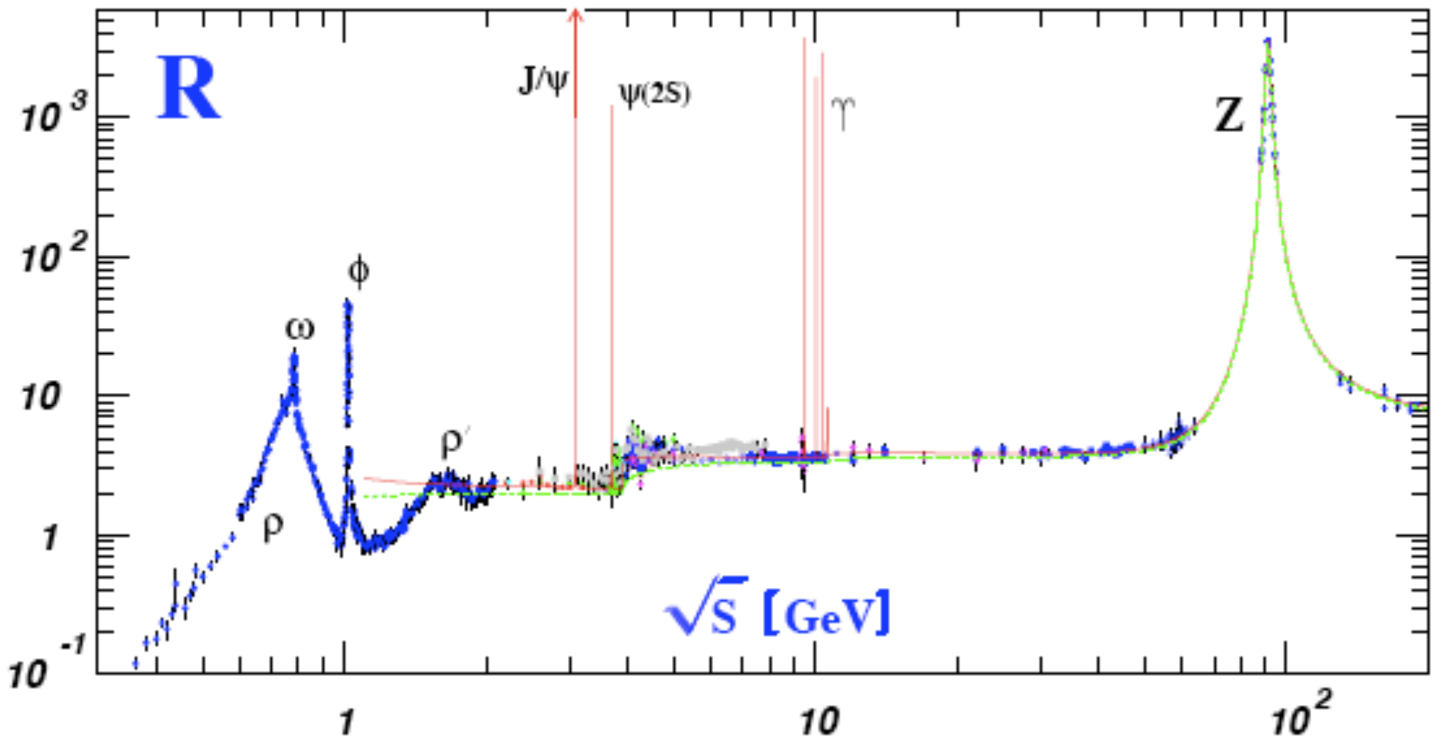
u, d, s, c: $R_0 = 3\,(q_u^{\,2} + q_d^{\,2} + q_s^{\,2} + q_c^{\,2}) = 10/3 = 3.3$

u, d, s, c, b: $R_0 = 3\,(q_u^{\,2} + q_d^{\,2} + q_s^{\,2} + q_c^{\,2} + q_b^{\,2}) = 11/3 = 3.7$

u, d, s, c, b, t: $R_0 = 3\,(q_u^{\,2} + q_d^{\,2} + q_s^{\,2} + q_c^{\,2} + q_b^{\,2} + q_t^{\,2}) = 5$

World Average on R

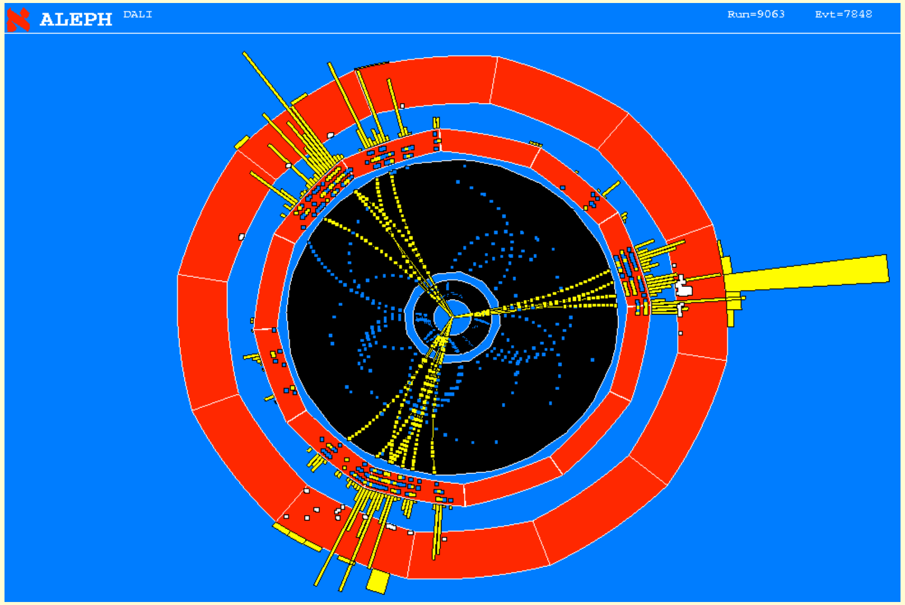
$N_C = 3$

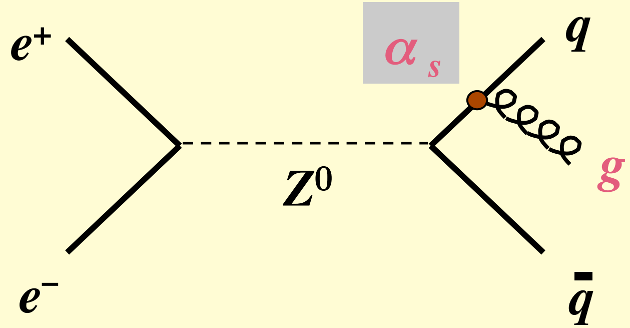


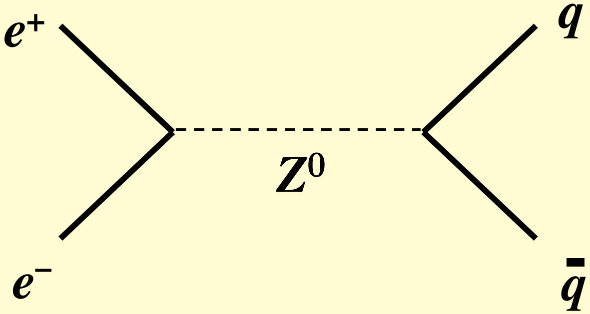
W.-M. Yao et al., Rev. Part. Phys., J. Phys. **G33** (2006) 1

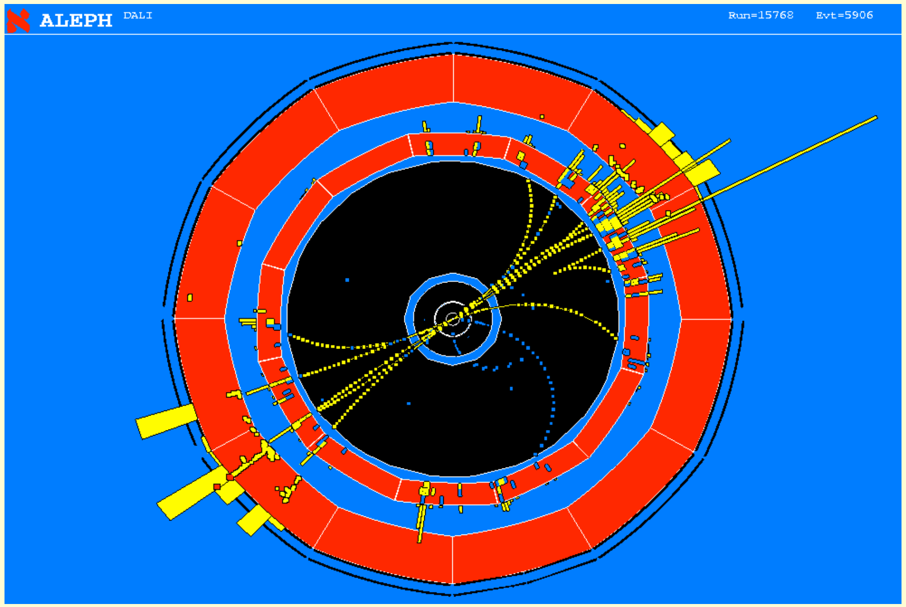
2-Jet and 3-Jet Events, α_s

The ratio between 3-jet and 2-jet events can be used to measure $\alpha_s = g_s/4\pi$.



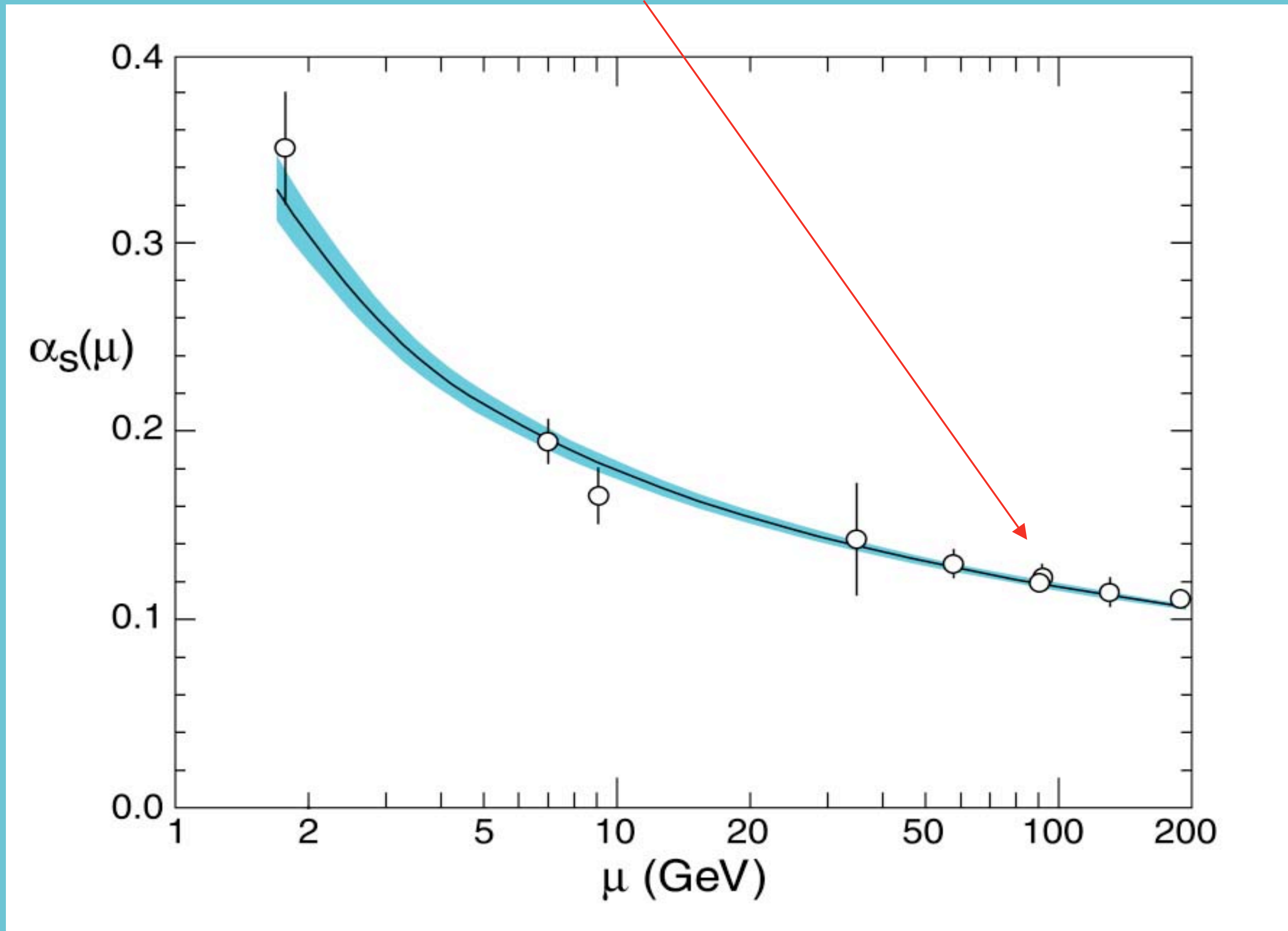






Running Coupling Constant

$\alpha_s(m_Z^2) = 0.118 \pm 0.002$



QCD Non-Abelian Gauge Symmetry

Global $SU(3)_C$ transformations in color space for q_f^{α}, a quark field of flavor f and color α :

$$q_f^\alpha \longrightarrow (q_f^\alpha)' = U^\alpha_\beta q_f^\beta \qquad\qquad UU^\dagger = U^\dagger U = 1 \quad det\ U = 1$$

The $SU(3)_C$ matrices can be written in the form: $U = exp\{i\frac{\lambda^a}{2}\theta^a\}$

θ^a … arbitrary parameters

$\lambda^a/2$ (a = 1, …, 8) … generators of the fundamental representation of $SU(3)_C$

λ^a … 3-dimensional Gell-Mann matrices $[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}] = i f^{abc}\frac{\lambda^c}{2}$ f^{abc} … *structure constants* (real, antisymm.)

Similar to QED, we require that the QCD Lagrangian be also invariant under local $SU(3)_C$ transformations $\theta^a = \theta^a(x)$ by using covariant derivatives:

$$\mathcal{D}^\mu q_f = [\partial^\mu + ig_s\frac{\lambda^a}{2}G^\mu_a(x)]\ q_f$$

g_s is the strong coupling constant.

Since there are 8 gauge parameters, we need 8 gluon fields (a=1,..,8): $G^\mu_a(x)$

QCD Gauge Transformations

The gauge transformation of the gluon fields is more complicated than the one obtained in QED for the photon, as the non-commutativity of the $SU(3)_C$ matrices gives rise to an additional term involving the gluon fields themselves.

$$G_a^\mu \longrightarrow (G_a^\mu)' = G_a^\mu - \frac{1}{g_s}\partial^\mu(\delta\theta_a) - f^{abc}\delta\theta_b G_c^\mu$$

Introduce field strengths, to build the gauge invariant kinetic term for the gluon fields:

$$\mathcal{D}^\mu q_f = [\partial^\mu + i g_s \tfrac{\lambda^a}{2} G_a^\mu(x)]\ q_f = [\partial^\mu + i g_s G^\mu(x)] q_f$$

$$G_a^{\mu\nu}(x) = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f^{abc} G_b^\mu G_c^\nu$$

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \sum_f \bar{q}_f^\alpha (i\gamma^\mu \partial_\mu - m_f) q_f^\alpha}$$

Decomposition of the QCD Lagrangian

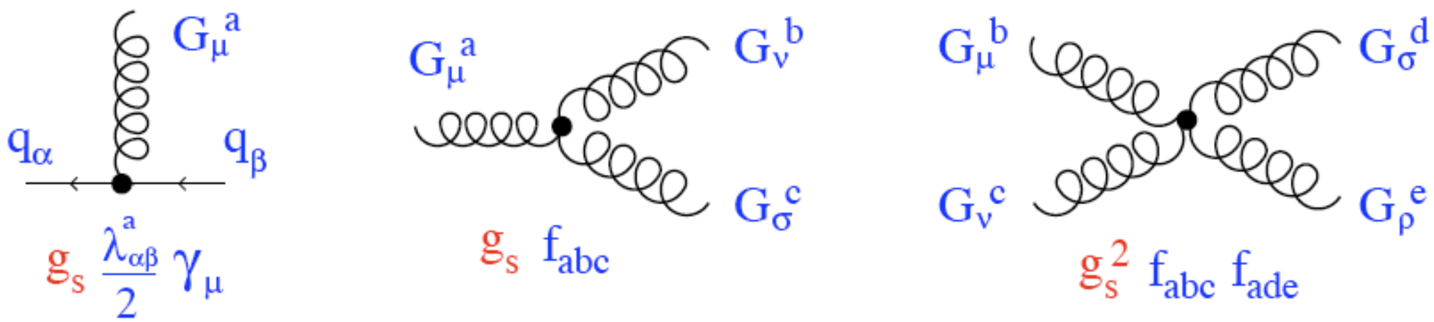
We can decompose $\mathcal{L}_{\rm QCD}$ into its different pieces:

$$\mathcal{L}_{QCD} = -\tfrac{1}{4}(\partial^\mu G^\nu_a - \partial^\nu G^\mu_a)(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu) + \sum_f \bar{q}^\alpha_f (i\gamma^\mu \partial_\mu - m_f) q^\alpha_f \qquad \text{(a)}$$

$$-g_s G^\mu_a \sum_f \bar{q}^\alpha_f \gamma_\mu \left(\tfrac{\lambda^a}{2}\right)_{\alpha\beta} q^\alpha_f \qquad \text{(b)}$$

$$+\tfrac{g_s}{2} f^{abc} (\partial^\mu G^\nu_a - \partial^\nu G^\mu_a) G^\mu_b G^\mu_c - \tfrac{g_s^2}{4} f^{abc} f_{ade} G^\mu_b) G^\mu_c G^d_\mu G^e_\nu \qquad \text{(c)}$$

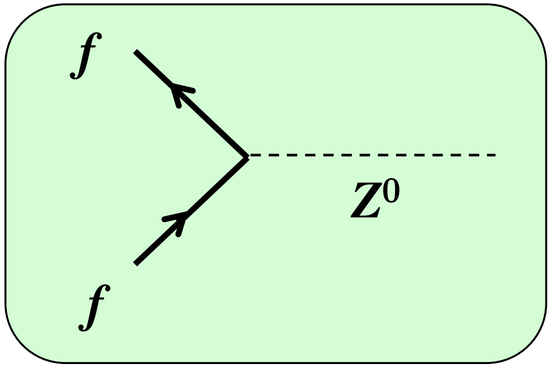
(a) Kinetic terms for the gluon and quark fields
(b) Color interaction between quarks and gluons
(c) Cubic and quartic gluon self-interactions



Electroweak Interaction

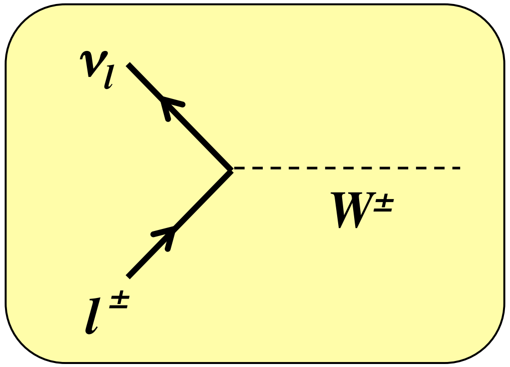
Low-energy information was sufficient to determine the structure of modern electroweak theory. The W and Z were introduced, and their masses correctly estimated before their experimental discovery.

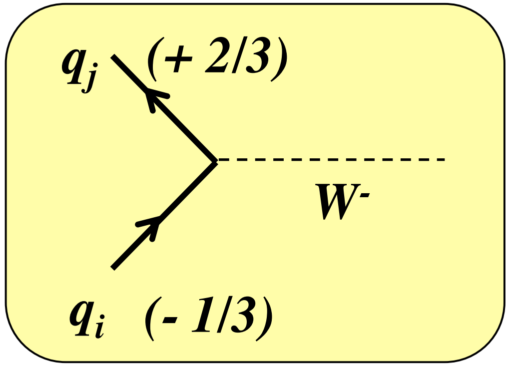
Neutral currents:



f … fermion (quarks, leptons - including neutrinos)

Charged currents:





l … e, μ, τ
q … quark
ν … neutrino

Charged Currents

Only left-handed fermions and right-handed antifermions couple to the W. Therefore, parity $\mathcal{P}$ and charge conjugation $\mathcal{C}$ (particle <-> antiparticle) are maximally violated. $\mathcal{CP}$ is still conserved.

The W couple to the fermion doublets, where the electric charges of the two fermion partners differ by one unit. The decay channels of the W⁻ are then:

$$W^- \to e^-\bar{\nu}_e, \mu^-\bar{\nu}_\mu, \tau^-\bar{\nu}_\tau, d'\bar{u}, s'\bar{c}$$

All fermion doublets couple to the W with the same universal strength.

The doublet partners of the u, c and t appear to be mixtures of the three charge -1/3 quarks, related through the unitary Cabibbo-Kobayashi-Maskawa matrix:

$$\boxed{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \mathbf{V_{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}}$$

M. Kobayashi
T. Maskawa
2008

The weak eigenstates are different from the mass eigenstates. V characterizes flavor mixing, e.g. V_{ud} specifies coupling of u to d $(d \rightarrow u + W^-)$.

$SU(2)_L \otimes U(1)_Y$

Gauge invariance was crucial for determining the right QED and QCD Lagrangians. For weak interactions it is more complicated, since we have several fermionic flavors and different properties for left- and right-handed fields. Moreover, the left-handed fields should appear in doublets, and the gauge bosons W and Z should be massive, since the weak interaction is of short range. If we want to include also electromagnetic interactions, we need an additional U(1) group. The obvious group to accommodate all this is:

$$\mathbf{SU(2)_L \otimes U(1)_Y}$$

L refers to left-handed fields, $Y_{(W)}$ is the weak hypercharge (naïve identification with electromagnetism does not work). For left-handed leptons $Y_W = -1$, and for right-handed leptons $Y_W = -2$.

Leptonic Sector of $SU(2)_L \otimes U(1)_Y$

SU(2)$_L$ doublet: $L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \frac{1}{2}(1-\gamma_5)\begin{pmatrix} \nu \\ e \end{pmatrix}$

Singlet: $R = e_R = \frac{1}{2}(1+\gamma_5)e$

Transformation under SU(2)$_L$: $U_L = exp\{i\alpha^a \frac{\sigma^a}{2}\}$ (*a* = 1,2,3)

$$L \rightarrow L' = e^{i\alpha^a \frac{\sigma^a}{2}} L, \qquad R \rightarrow R' = R$$

and under U(1)$_Y$: $L \rightarrow L' = e^{i\beta \frac{Y_L}{2}} L, \qquad R \rightarrow R' = e^{i\beta \frac{Y_R}{2}} R$

Global transformations under $\mathbf{SU(2)_L \otimes U(1)_Y}$ in flavor space:

$$L' = e^{i\beta \frac{Y_L}{2}} U_L L$$
$$R' = e^{i\beta \frac{Y_R}{2}} R$$

Leptonic Sector of $SU(2)_L \otimes U(1)_Y$

We require the Lagrangian to be invariant under local gauge transformations [$\alpha_i = \alpha_i\ (x)$, $\beta = \beta\ (x)$] and introduce covariant derivatives as in QED. Since there are 4 gauge parameters, 4 different gauge bosons are needed:

$$\mathcal{D}_\mu = \partial_\mu + ig\frac{\sigma^a}{2}W^a_\mu + ig'\frac{Y}{2}B_\mu$$

Explicitly for L and R lepton states:

$$\mathcal{D}_\mu L = [\partial_\mu + ig\frac{\sigma^a}{2}W^a_\mu(x) + ig'\frac{Y_L}{2}B_\mu(x)]L$$
$$\mathcal{D}_\mu R = [\partial_\mu + ig'\frac{Y_R}{2}B_\mu(x)]R$$

We have the correct number of gauge bosons, since we need the photon and 3 intermediate vector bosons $W^\pm$, Z.

Lagrangian of $SU(2)_L \otimes U(1)_Y$

The complete electroweak Lagrangian is actually quite complicated, impossible to derive in the timeframe of this lecture.

$$\mathcal{L}_{EW} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_S + \mathcal{L}_Y + \mathcal{L}_{fix} + \mathcal{L}_{gh}$$

$\mathcal{L}_G$ pure gauge-field, $\mathcal{L}_F$ fermion-gauge field, $\mathcal{L}_S$ scalar,
$\mathcal{L}_Y$ fermion-scalar (Yukawa), $\mathcal{L}_{fix}$ gauge fixing, $\mathcal{L}_{gh}$ ghosts

Kinetic term for the gauge fields, which also includes self-interactions of the gauge bosons:

$$\mathcal{L}_G = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W_i^{\mu\nu}$$

Field strengths: $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

Note: No mass term allowed, since it would violate gauge symmetry by mixing left- and right-handed fields.

Example for fermionic mass term: $\mathcal{L}_m = -m\bar{\psi}\psi = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$

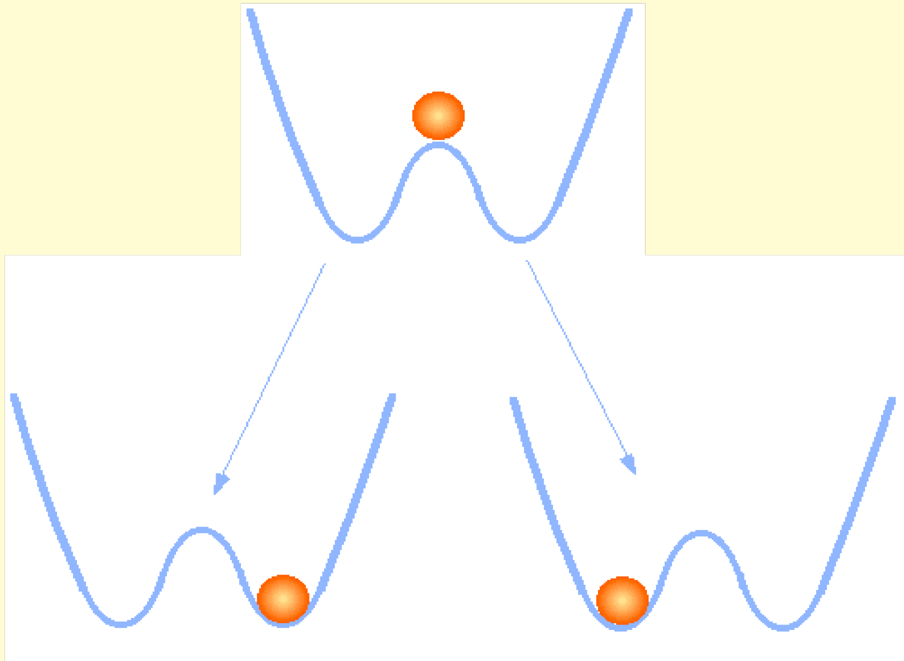
Absence of mass is fine for the photon, but we need heavy vector bosons in order to get short-range weak interactions!

Spontaneous Symmetry Breaking

In order to generate masses, we need to break the gauge symmetry. How is this possible with a symmetric Lagrangian (which is also needed to preserve renormalizability of a theory)?

-> By choosing a Lagrangian that is invariant under a group of transformations, and that has a degenerate set of states with minimal energy.

The particle has to choose a state with minimal energy -> symmetry is broken (actually hidden).



Y. Nambu 2008

Goldstone Theorem

Consider a complex scalar field $\phi(x)$, with a Lagrangian invariant under global phase transformations of $\phi(x)$ and with potential V:

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi), \qquad V(\phi) = \mu^2 \phi^\dagger \phi + h(\phi^\dagger \phi)^2$$

In order to have a ground state, the potential should be bounded from below, i.e. $h > 0$. For the quadratic term there are 2 possibilities:

$\mu^2 > 0$: The potential has only the trivial minimum $\phi(x) = 0$. It describes a massive scalar particle with mass μ and quartic coupling h.

$\mu^2 < 0$: The minimum is obtained for those field configurations with:

$$|\phi_0| = \sqrt{\frac{-\mu^2}{2h}} = \frac{v}{\sqrt{2}} > 0$$

Goldstone Theorem

Due to the U(1) phase invariance of the Lagrangian, there is an infinite number of degenerate states of minimum energy:

$$\phi_0(x) = \frac{v}{\sqrt{2}} e^{i\theta}$$

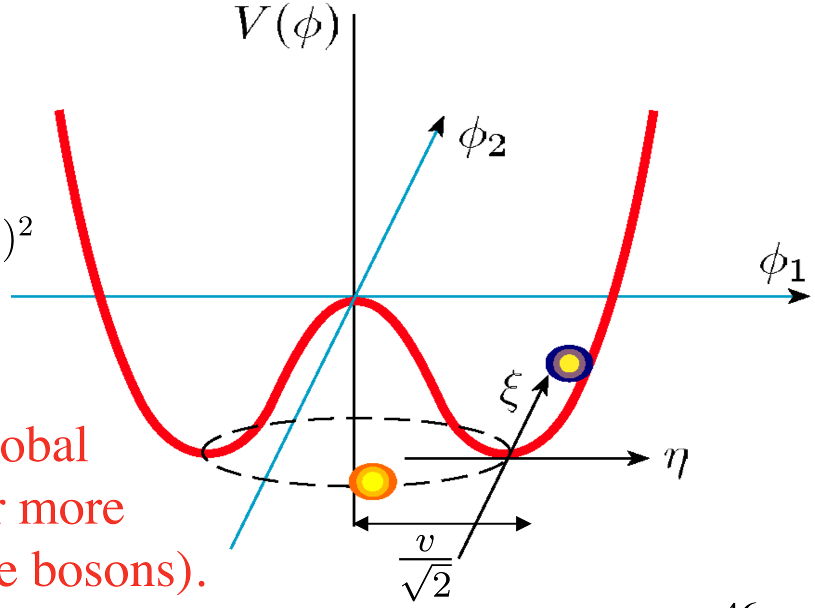
If we choose a particular solution as the ground state, e.g. $\theta = 0$, the symmetry gets spontaneously broken. We can parameterize the excitations over the ground state as:

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \eta(x) + i\xi(x)]$$

$$V(\phi) = V(\phi_0) - \mu^2\eta^2 + hv\eta(\eta^2 + \xi^2) + \frac{h}{4}(\eta^2 + \xi^2)^2$$

η describes a massive state of mass $-2\mu^2$,
ξ a massless state.

Goldstone theorem: SSB of a continuous global symmetry is always accompanied by one or more massless scalar (spin 0) particles (Goldstone bosons).



The Higgs Sector

Obviously, the Goldstone theorem did not solve our problem of massive gauge bosons. However, what happens if we have a *local* gauge symmetry?
We try to introduce a new doublet of complex, scalar fields with weak hypercharge $Y_\Phi = 1$ to accomplish the breaking of electroweak symmetry, leaving the electromagnetic gauge subgroup $U(1)_{em}$ unbroken:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$$

It is coupled to the gauge fields through the scalar Lagrangian, which is invariant under local $\mathbf{SU(2)_L \otimes U(1)_Y}$ transformations:

$$\mathcal{L}_S = (\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi - V(\Phi) = (\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi - \mu^2 \Phi^\dagger \Phi - h(\Phi^\dagger \Phi)^2 \qquad h > 0, \mu^2 < 0$$

$$\mathcal{D}_\mu = \partial_\mu + ig\frac{\sigma^a}{2} W_\mu^a + ig'\frac{Y_\Phi}{2} B_\mu$$

The potential $V(\Phi)$ is constructed in such a way that Φ has a non-vanishing vacuum expectation value:

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

Higgs-Kibble Mechanism

$\Phi(x)$ can be written in the form:

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ (v + H(x) + i\chi(x))/\sqrt{2} \end{pmatrix}$$

The components $\phi^+(x)$, H, χ have vacuum expectation values 0. The local $SU(2)_L$ invariance of the Lagrangian allows to rotate away ("unitary gauge") any dependence on ϕ^+ and χ. This means that these are unphysical, they correspond to 3 "ghosts" or Goldstone bosons (remember, ϕ^+ is complex, with 2 real parameters).
In this particular gauge, the **Higgs field** has the simple form:

$$\boxed{\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}}$$

The real field $H(x)$ describes physical, neutral particles with mass $m_H = \mu\sqrt{2}$.
Vacuum expectation value: v = 246 GeV.

The Higgs Boson

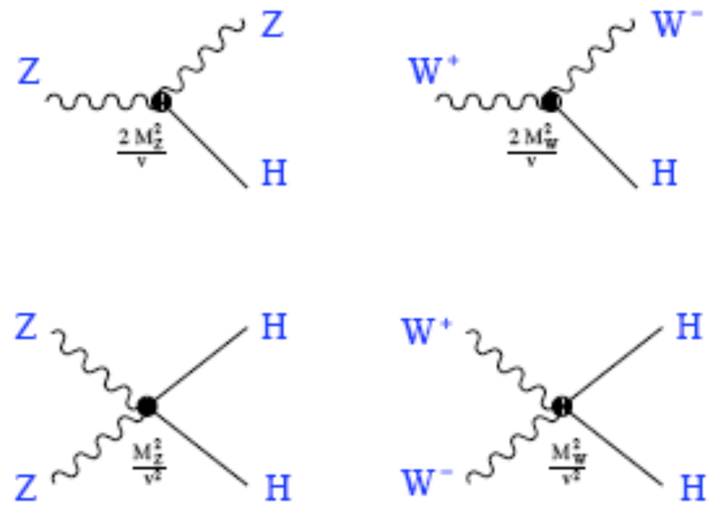
The scalar Lagrangian $\mathcal{L}_S = (\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi - \mu^2 \Phi^\dagger \Phi - h(\Phi^\dagger \Phi)^2$ has introduced a new scalar particle, the Higgs boson H. In terms of the physical fields, in the unitary gauge, $\mathcal{L}_s$ takes the form:

$$\mathcal{L}_S = \frac{1}{4} h v^4 + \mathcal{L}_H + \mathcal{L}_{HG^2}$$

$$\mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{1}{2} m_H^2 H^2 - \frac{m_H^2}{2v} H^3 - \frac{m_H^2}{8v^2} H^4$$

$$\mathcal{L}_{HG^2} = m_W^2 W_\mu^\dagger W^\mu \{1 + \frac{2}{v} H + \frac{H^2}{v^2}\} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \{1 + \frac{2}{v} H + \frac{H^2}{v^2}\}$$

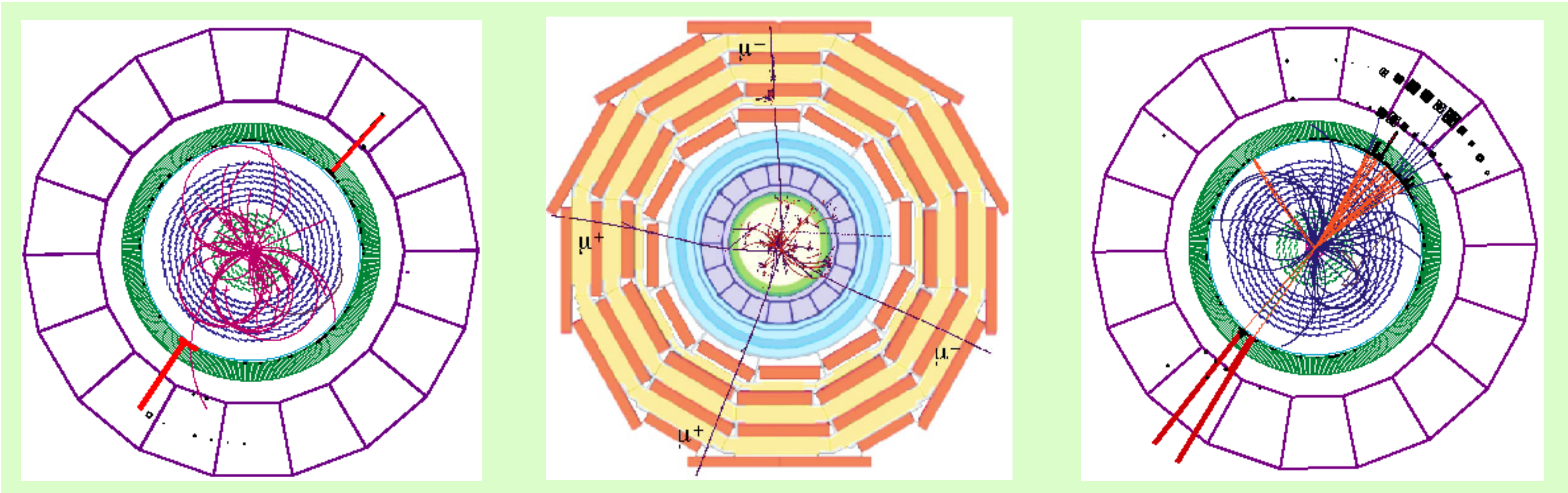
Higgs couplings to the gauge bosons:

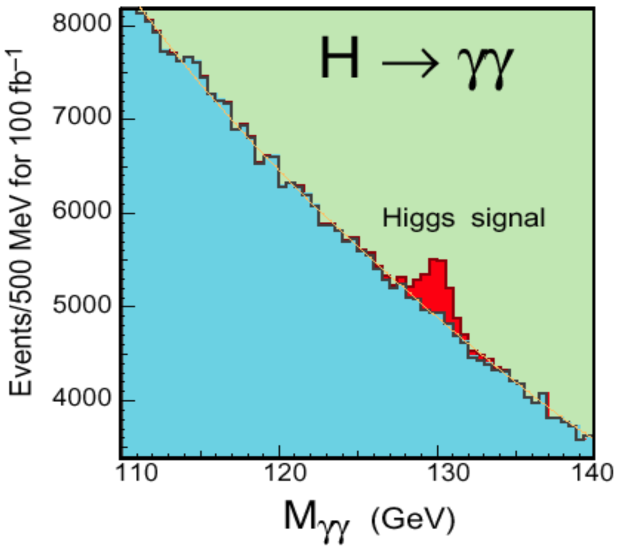


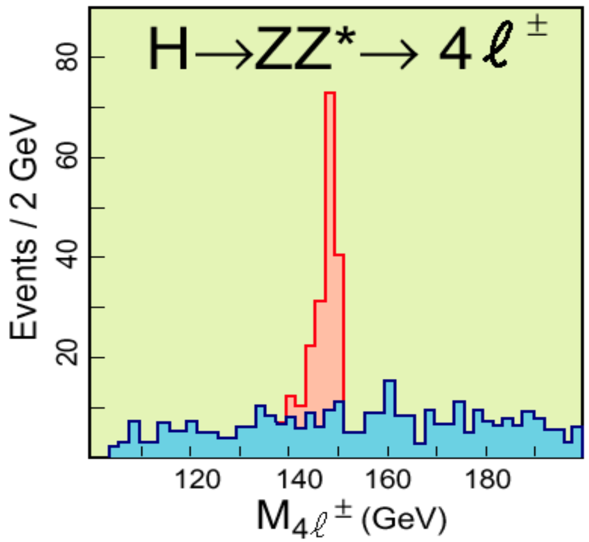
Higgs in CMS

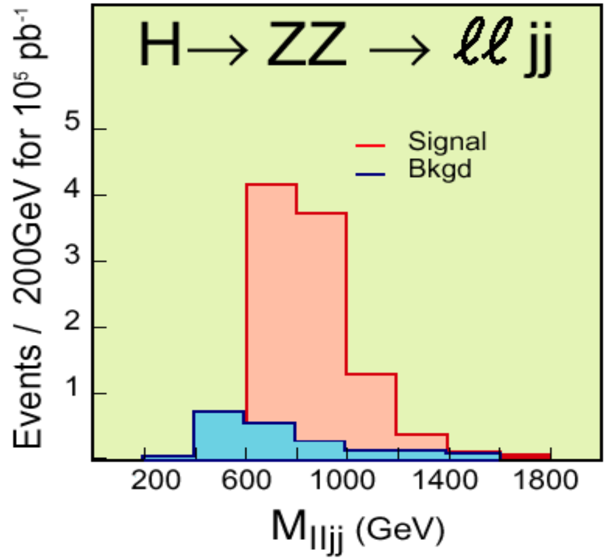
Higgs in CMS

Higgs Boson at LHC (CMS Experiment)









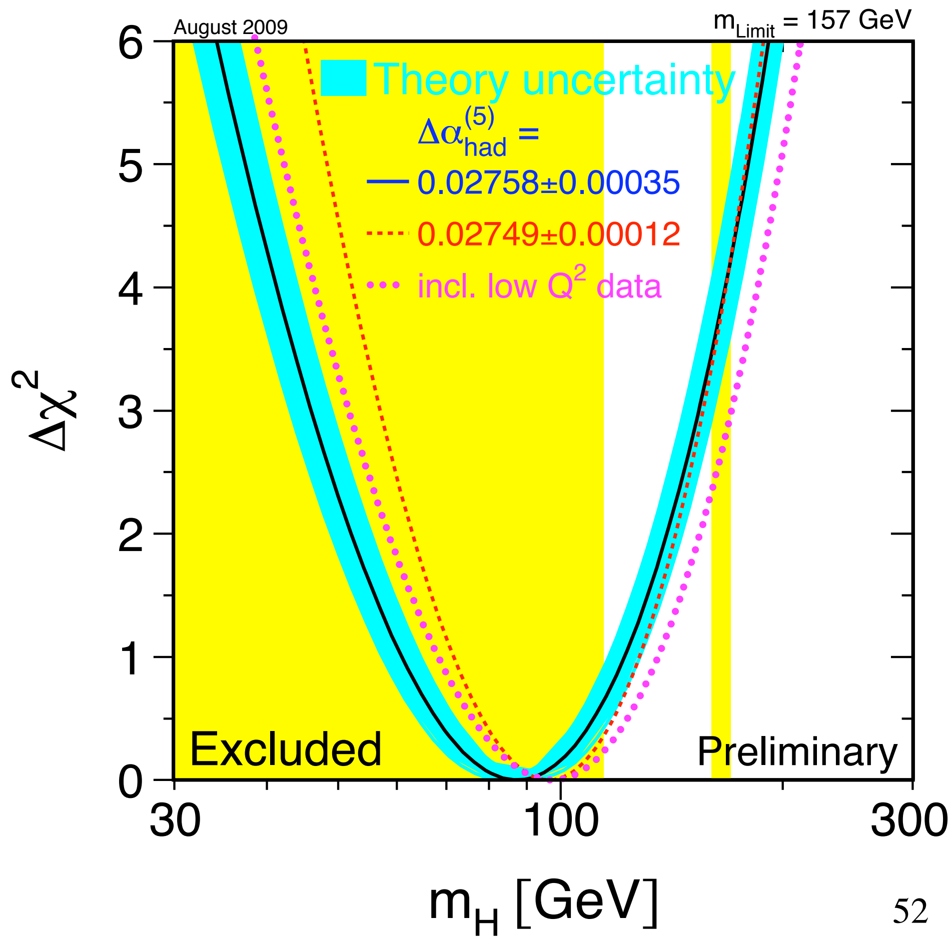
Higgs Boson Mass Limits

Direct search at LEP ended 2000. Result: $m_H > 114.4$ GeV/c^2 @ 95% c.l.

From 'precision electroweak fits' (LEP, SLD, CDF, D0):

Preferred value:
$m_H = (87 +35 - 26)$ GeV/c^2

$m_H < 157$ GeV/c^2 @ 95% c.l.



W, Z, Photon, Electroweak Unification

The covariant derivative $\mathcal{D}_\mu = \partial_\mu + ig\frac{\sigma^a}{2}W^a_\mu + ig'\frac{Y_\Phi}{2}B_\mu$ couples the scalar doublet to the $\mathbf{SU(2)_L \otimes U(1)_Y}$ gauge bosons. In the unitary gauge the kinetic part of the scalar Lagrangian takes the form:

$$(\mathcal{D}_\mu\Phi)^\dagger\mathcal{D}^\mu\Phi \to \frac{1}{2}\partial_\mu H\partial^\mu H + (v+H)^2\{\frac{g^2}{4}W^\dagger_\mu W\mu + \frac{g^2}{8cos^2\theta_W}Z_\mu Z^\mu\}$$

with the following transformation from the fields $W_\mu{}^a$, B_μ to the physical $W^\pm$ and Z fields:

$$W_\mu = \frac{W^1_\mu + iW^2_\mu}{\sqrt{2}} \qquad W^\dagger_\mu = \frac{W^1_\mu - iW^2_\mu}{\sqrt{2}} \qquad \text{W}^-, \text{W}^+$$

$$A_\mu = cos\theta_W B_\mu + sin\theta_W W^3_\mu \qquad \text{Photon } \gamma$$

$$Z_\mu = -sin\theta_W B_\mu + cos\theta_W W^3_\mu \qquad \text{Z}^0$$

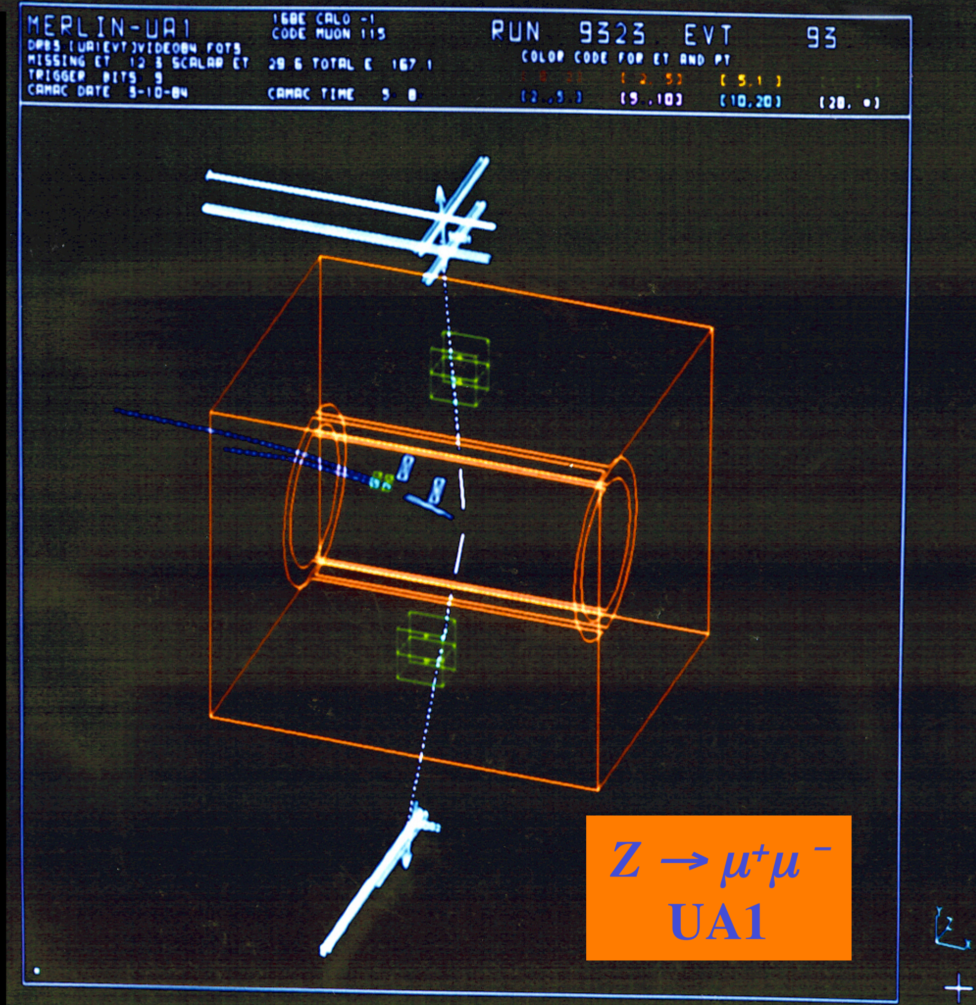
The vacuum expectation value of the neutral scalar has generated a quadratic term for the W and Z, these bosons have acquired mass:

$$\boxed{m_Z cos\theta_W = m_W = \frac{1}{2}gv} \qquad \boxed{m_Z = \frac{1}{2}\sqrt{g^2+g'^2}\ v}$$

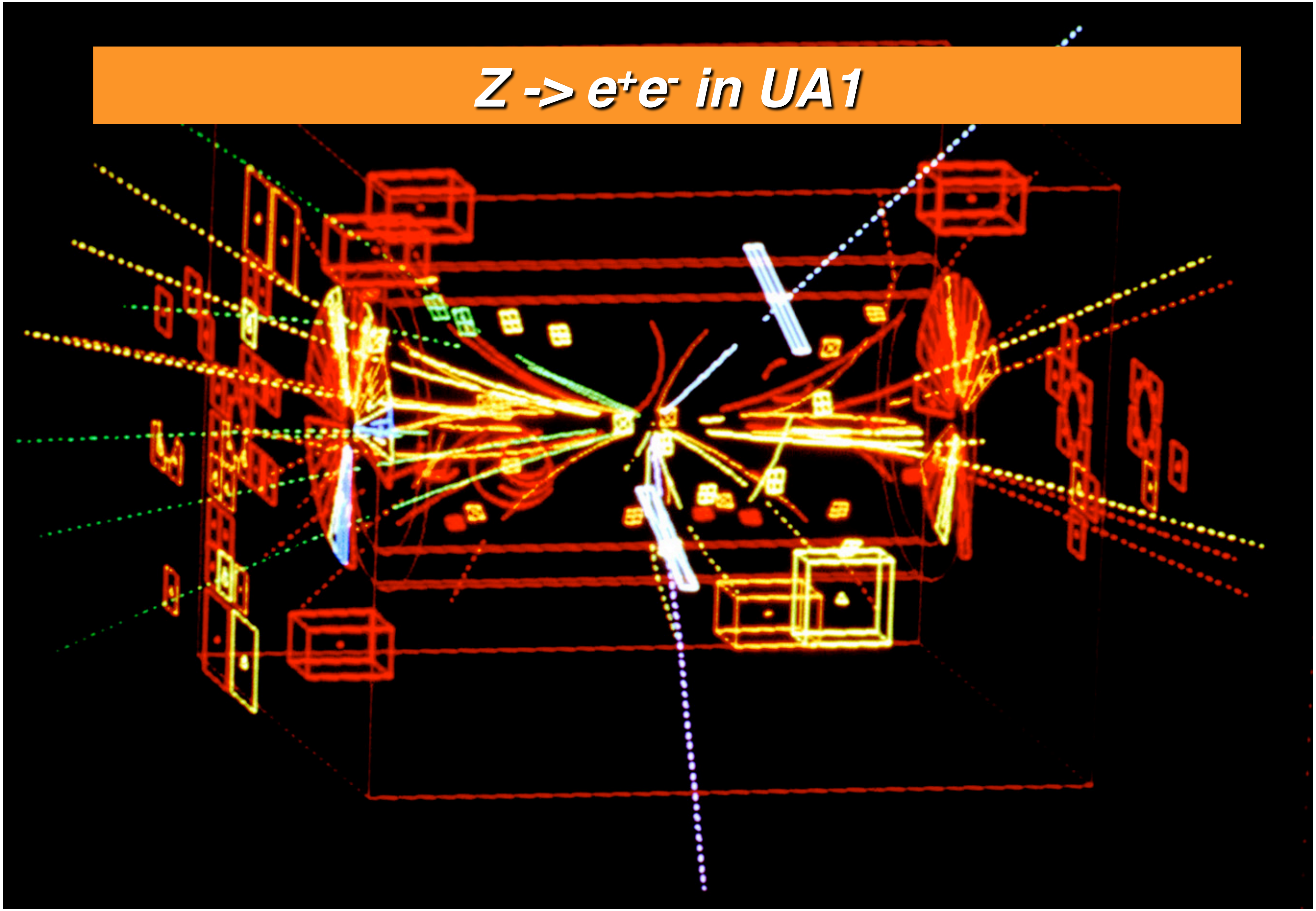
θ_W … Weinberg angle ($\theta_W \approx 28^0$, $\sin\theta_W \approx 0.23$) $\quad e = \frac{gg'}{\sqrt{g^2+g'^2}} \quad \alpha = \frac{e^2}{4\pi}$

Discovery of the W and Z 1983

Experiments UA1 and UA2 at the CERN Super-Proton-Antiproton Collider.
Nobel Prize for C. Rubbia and S. van der Meer 1984.



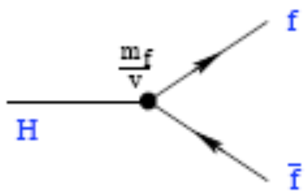
Z -> e^+e^- in UA1



Fermion Masses

We need not only masses for the W and Z, but also fermion masses (at least for the charged fermions in the classical Standard Model). A fermionic mass term of the form $\mathcal{L}_m = -m\bar{\psi}\psi = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$ is not allowed since it violates gauge symmetry. Since we have introduced an additional scalar doublet into the model, we can write the following gauge-invariant Yukawa Lagrangian describing fermion-scalar coupling ($f = u, d, e, \ldots$):

$$\mathcal{L}_Y = -\sum_f m_f \bar{\psi}_f \psi_f - \sum_f \frac{m_f}{v} \bar{\psi}_f \psi_f H$$



$$m_f = g_f \frac{v}{\sqrt{2}}$$

$g_f \ldots$ Yukawa couplings

Yukawa interactions between the massive fermions and the physical Higgs field occur with coupling constants proportional to the fermion masses.

Conclusions

The Standard Model works remarkably well, up to O(100 GeV).
Some quantities are determined at the 0.1% level!

Nevertheless:
Neutrino masses are not accommodated in the classical Standard Model.
There is the hierarchy problem (stability of the mass of the Higgs particle).
There is no unification of coupling constants at very high energies.
Gravitation is not included at all.
There is no explanation for dark matter or dark energy.
We do not know what happened just after the Big Bang.

Therefore:
Particle physicists, astrophysicists and cosmologists need to collaborate in the near future to find the proper extensions to the Standard Model. We have excellent tools, such as accelerators, space probes, ground-based telescopes, underground laboratories. and event nuclear reactors. Precision experiments at ultra-low energies can also contribute.

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